Written Exam at the Department of Economics Summer 2018

# Micro III

Final Exam

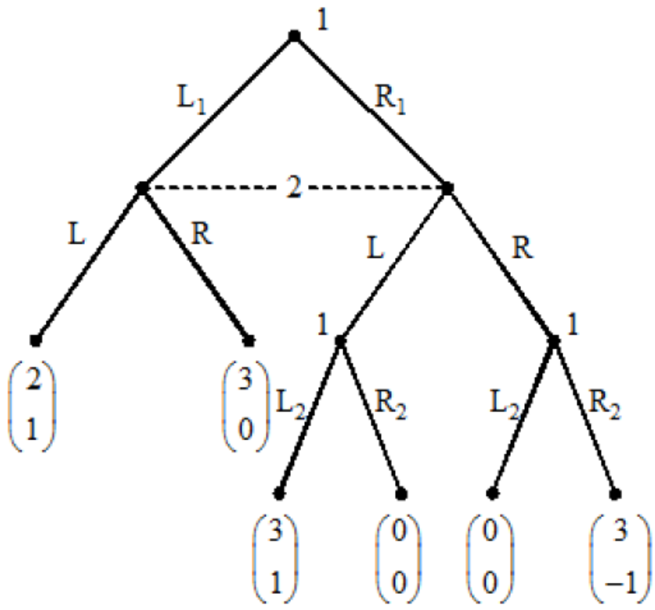
August 22, 2018

(2-hour closed book exam)

Answers only in English.

**This exam question consists of 3 pages in total (including the current page).**

1. Consider the following game $G$




   (a) Briefly explain whether $G$ is a game of perfect or imperfect information (1 sentence).

   (b) How many proper subgames are there in $G$ (i.e. not including the game itself)? How many strategies does Player 1 and Player 2 have in this game?

   (c) Would your answer to (a) change if we instead considered a game which was identical to $G$ in all respects, expect that Player 1 could not observe whether Player 2 chose $L$ or $R$? What about your answer to (b)? *Please write just 'Yes' or 'No', for each of these two subquestions.*

   (d) Solve for the unique pure strategy subgame perfect Nash equilibrium of $G$.

   (e) Solve for all pure strategy Nash equilibria of $G$. If there are multiple Nash equilibria, then pick one Nash equilibrium which is not subgame perfect, and explain in words why this is the case (2-3 sentences).

2. Consider a static game $F$ where two firms produce a homogeneous good and compete in quantities. Firm 1 and Firm 2 both produce at zero cost. Let $q_i$ denote the quantity produced by Firm $i \in \{1, 2\}$. Given $q_1$ and $q_2$, the market price is $p = 3 - q_1 - q_2$. Both firms choose quantities simultaneously, and maximize profits.

   (a) Solve for the Nash equilibrium of this game. What profits do Firm 1 and Firm 2 earn in equilibrium? What profits would Firm 1 and Firm 2 earn if they instead each produced half of the monopoly quantity (i.e. half of the quantity that maximizes total industry profits)?

   Now consider a dynamic game, with infinite time horizon, where Firm 1 and Firm 2 play the stage game $F$ in periods $t = 1, 2, 3, \ldots$. You can assume that both firms discount future payoffs with factor $\delta \in (0, 1)$.

   (b) Consider a candidate subgame perfect Nash equilibrium where, on the equilibrium path, each firm produces half of the monopoly quantity in each period. Write down trigger strategies for Firm 1 and Firm 2 that could potentially sustain such an equilibrium. Write down an inequality which implicitly defines the values of $\delta$ for which neither firm has an incentive to deviate from their equilibrium strategy (you do **not** need to explicitly solve this inequality to isolate $\delta$). Briefly give some intuition as to why the value of the discount factor affects the incentive to deviate (2-3 sentences).

(c) Now consider a candidate subgame perfect Nash equilibrium where, on the equilibrium path, firms produce the following quantities: in odd periods, $t = 1, 3, 5, \ldots$, Firm 1 produces the monopoly quantity and Firm 2 produces nothing; and in even periods, $t = 2, 4, 6, \ldots$, Firm 2 produces the monopoly quantity and Firm 1 produces nothing. Write down trigger strategies for Firm 1 and Firm 2 that could potentially sustain such an equilibrium. Write down two inequalities which implicitly define the values of $\delta$ for which neither firm has an incentive to deviate from their equilibrium strategy (you do **not** need to explicitly solve these inequalities to isolate $\delta$). *Hint 1: think about what is a firm's best reply in a period where the other firm produces zero, and in a period where the other firm produces the monopoly quantity. Hint 2: you may use the fact that* $1 + \delta^2 + \delta^4 + \ldots = \frac{1}{1-\delta^2}$.

(d) Look back at the inequalities you derived in parts (b) and (c). Can you say something about whether the firms find it easier to sustain collusion if they each produce half the monopoly quantity in each period (as in (b)) or if they take turns each producing the monopoly quantity and zero (as in (c)) (3-4 sentences)? If so, briefly give some intuition (3-4 sentences). *Please attempt this question even if you did not successfully complete the earlier parts.*

3. A firm is hiring a worker. Workers are characterized by their type $\theta$, which measures their ability. There are two worker types: $\theta \in \{\theta_L, \theta_H\}$. Nature chooses the worker's type, with $\mathbb{P}(\theta = \theta_H) = p$ and $\mathbb{P}(\theta = \theta_L) = 1 - p$. Assume $p \in (0, 1)$. The worker observes his own type, but the firm does not.

The worker can choose his level of education: $e \in \mathbb{R}^+$. The cost to him of acquiring this education is

$$c_\theta(e) = \frac{e}{\theta}$$

Education is observed by the firm, who then forms beliefs about the worker's type: $\mu(\theta|e)$. We assume that the marginal productivity of a worker is equal to his ability $\theta$ and that the firm is in competition such that it pays the marginal productivity: $w(e) = \mathbb{E}(\theta|e)$. Thus, the payoff to a worker conditional on his type and education is

$$u_\theta(e) = w(e) - c_\theta(e)$$

Suppose for this exercise that $\theta_H = 4$ and $\theta_L = 2$.

(a) In a separating equilibrium, the low-ability worker chooses education level $e_L$ and obtains wage $w_L = w(e_L)$. Is it possible that $e_L > 0$? Explain briefly (max. 3 sentences).

(b) Find a separating pure strategy Perfect Bayesian Equilibrium where the two types choose education levels $e_L$ and $e_H$, respectively, and the low ability type is indifferent between choosing $e_L$ and $e_H$. Assume that off the equilibrium path, the firm assigns zero probability to the worker being type $\theta_H$.

(c) Find a pooling pure strategy Perfect Bayesian Equilibrium in which both types choose education level $\bar{e}$, and the low ability type is indifferent between choosing $e = 0$ and $e = \bar{e}$. Assume that off the equilibrium path, the firm assigns zero probability to the worker being type $\theta_H$. Does the pooling equilibrium you found satisfy Signaling Requirement 6 (*'equilibrium domination'*)? You can show this either graphically or algebraically.